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MODELLING AND OPTIMIZING MULTIPLE ATTRIBUTE DECISIONS BY USING FUZZY SETS

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Abstract. *The purpose of this paper is to present a coherent perspective of modeling and optimizing multiple attribute decisions by using fuzzy sets. In management practice we face most of the time the situation in which a problem have several possible solutions and each solution can be analyzed using multiple criteria models. In the same time, in real life decision making process there is a given level of uncertainty which makes difficult a clear cut analytical analysis. The object of this article is to build a model approach for making multiple criteria decision using fuzzy sets of objects. Elaborating multiple attribute decisions involves performing an assessment and selecting from a given and finite set of possible alternative courses of action in the presence of a given and finite, and usually conflicting set of attributes and criteria.*

Keywords: decision making, fuzzy sets, modeling, multiple criteria optimization.

1. General considerations

Elaborating decisions is a natural element of our daily lives. At organizational level, it is quite obvious that efficiency must have priority, obtaining economic-financial performances, expressed with the help of some indicators as a natural consequence of its managerial, general and specific performances. The main problem resides in the fact that most decisional problems have different and multiple – hence conflicting – criteria or attributes. Periodically, in the numerous specialized magazines of various fields of study, we come across diverse methodologies and their applications.

The diversity of problems we face daily in our professional activity or personal life suggests that it is possible to delimit these problems into two very wide general categories, namely:

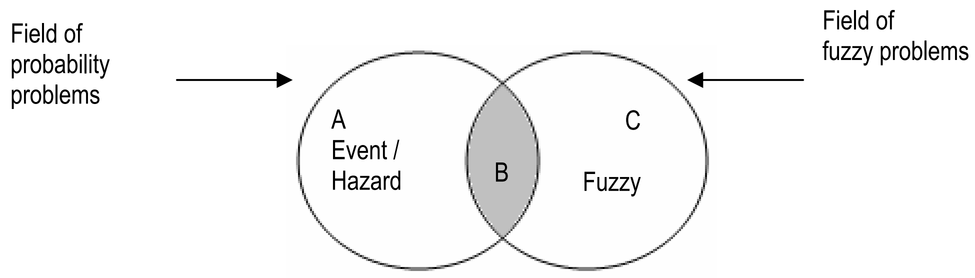
a) Multiple attribute or multiple criteria problems.
b) Multiple objective problems.

Approaching the act of decision making a practical perspective, we find that multiple attribute, or multiple criteria, problems are associated with issues that have a predetermined number of candidate alternatives. The decision maker must achieve or express an option, i.e. make a scale of preferences based on a finite number of alternative courses of action.

Conversely, multiple objective decisions are not associated with problems where we have a predetermined number of alternatives. The decision maker's main goal or aim is to project the most promising candidate alternative with respect to the finite resources available to him.

Our existence is continuously marked by incertitude. The research and the approach by which it is attempted to model incertitude in decision taking is based on the theory of probabilities, which holds the stochastic analysis or analysis of decisional situations. The second approach holds and retains aspects pertaining to human behavioral subjectivity. It is suggested by Efstathiou (Efstathiou,1979), Dubois and Prade (Dubois and Prade,1982) that the stochastic method of decision, such as the statistic analysis of decision, does not measure the precision, the exactness of human behavior; rather, the method represents a way to model incomplete knowledge regarding external environment that envelops human existence. On the other hand, the theory of fuzzy sets of objects represents a perfect instrument to model incertitude (or imprecision) that results from mental phenomena that are neither accidental nor stochastic.

The two groups of problems are presented in figure no 1, where C represents our field of interest. Certainly, it is hard to involve human existence in the process of analyzing decisions. A nation-wide approach to decision taking will taking into consideration human subjectivity more than the use of objective probability measures. This attitude versus the incertitude of human behavior leads us to the object of a new field of analysis of decisions, i.e. elaborating fuzzy decisions.



(Event / Hazard and Fuzzy)

Figure 1. Field of probability problems and fuzzy

The object of this article is to build a model approach for making multiple criteria decision using fuzzy sets of objects.

Elaborating multiple attribute decisions involves performing an assessment and selecting from a given and finite set of possible alternative courses of action in the presence of a given and finite, and usually conflicting set of attributes and criteria.

Such a problem of multiple attribute decision can be expressed concisely in a matrix form, e.g. like the one presented in figure no. 2.

	A_i \ C_j	C_1	C_2	.....	C_j	....	C_n
	A_1	a_{11}	a_{12}	.....	a_{1j}	....	a_{1n}
	A_2	a_{21}	a_{22}	.....	a_{2j}	....	a_{2n}
(D)$\Rightarrow$	$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
	A_i	a_{i1}	a_{i2}	.....	a_{ij}	....	a_{in}
	$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
	A_m	a_{m1}	a_{m2}	.....	a_{mj}	....	a_{mn}

Figure 2. Matrix of decisional situation

where: A_i ($i = \overline{1,m}$)=> the set of possible courses of action

C_j ($j = \overline{1,n}$)=> the set of attributes of the assessment criteria (usually conflicting)

a_{ij}=> the set of candidate alternative consequences in the presence of criteria C_j

D=> the decision maker

There are numerous studies in the specialized literature on solving multiple attribute problems. However, two researchers, Hwang and Yoon, offer and full and systematic research of the methods which lead to solving decisional multiple attribute problems (Hwang and Yoon, 1981).

It is not odd that sometimes a_{ij} (respectively, assessment of consequences) cannot be assessed or measured precisely. Imprecision has various sources, of which the most common are:

- *Intangible* (non-measurable) *information*. For example, the decision concerning the selection of a bus by a firm of public transportation: the price is easy to identify and determine; conversely, safety in exploitation, comfort, design, ease of handling, cannot be quantified and measured exactly. The attributes or criteria: comfort, design, safety and ease of handling are performances that are expressed by intangible terms, such as: fine, beautiful, big, etc. In reality, this is qualifying, appreciative, information expressing mostly a person's subjective judgment or reasoning, i.e. it is dependant, consequently fuzzy.
- *Incomplete information*. With the help of a measuring device, the fuel consumption of a vehicle can be measured approximately and relatively, to be 5 liters / 100 km, but this does not mean it is exactly 5 liters / 100 km. Such information belongs to the category of fuzzy information, because it is incomplete.
- *Information that is hard or impossible to obtain* (sometimes there is a shortage of correct and exact information). Sometimes clear and precise information and data can definitely be obtained, but the cost of getting it is

too high, in which case the decision maker gives up obtaining them in their exact form; however, he wants to obtain at least estimations or approximations of the definite information and data. On other occasions, the data and information can be painful or delicate by their absence (e.g. a state secret, top secret, a firm secret, the value of an individual's bank account, the age of certain members of the target audience in marketing decisions, etc., a situation in which such information or data are expressed approximatively, estimatively, by literary description. In this case, too, we are dealing with vague information, of the fuzzy type.

- *Partial knowledge or ignorance*. A number of ambiguities may appear about the conditions regarding certain phenomena or parts of a phenomenon, and hence, only part of the facts will be known, which again makes the information fuzzy.

The classical methods and models for optimizing multiple attribute decisions cannot actually handle, control and approach - are not compatible with - problems with such inaccurate or vague information. To solve and overcome these difficulties, the theory of fuzzy was called upon, introduced initially by Zadeh (Zadeh, 1965). A fuzzy set of objects is defined by Zadeh as follows:

Let U be a classical (or ordinary) set of objects, called the universe, whose elements are generically symbolized as "x", i.e. U={x}. A fuzzy set of objects A in U is characterized by a membership function $u_A(x)$, which associates with each element in U a real number in the interval [0,1]. The fuzzy set, A, is usually denoted by a set of pairs:

$$A=\{ x; u_A(x), x \in U\} \tag{1}$$

For an ordinary set, A,

$$u_A(x) = \begin{cases} 1 \text{ dacă } x \in A \\ 0 \text{ dacă } x \notin A \end{cases} \tag{2}$$

When U is a finite set $\{x_1, \ldots , x_n\}$, the fuzzy set for U can also be represented following Zadeh (Zadeh, 1973) or Dubois and Prade (Dubois and Prade, 1980)

$$A = \sum_{i=1}^{n} x_i / u_A(x_i)\,. \tag{3}$$

When U is an infinite set, the fuzzy set can be reprezented:

$$A = \int_x x / u_A(x) \tag{4}$$

Example 1 (Chen and Hwang, 1992): Let U={Ken; John; Allen; Peter} be a finite set. If the evaluator is a girl, the fuzzy set "handsome boys" may be characterized as:

A={(Ken, 0.7); (John, 0.2); (Allen, 0.8); (Peter, 0.6)}

or

A=Ken/0.7+John/0.2+Allen/0.8+Peter/0.6

Example 2 (Zimmermann, 1983): Let U={10; 20; 30; 40; 50; 60; 70; 80; 90; 100}, be the possible speed (mph) at which cars can run over a long distance. Then the fuzzy set "comfortable speed for long distance travel" may be defined by an individual person as:

A={(30, 0.7); (40, 0.75); (50, 0.80); (60, 0.80); (70, 1.0); (80, 0.8); (90, 0.3)}

Note that x=10, 20 and 100, are viewed as "absolutely uncomfortable cruising speed", i.e. the degree of comfort is zero. They are omitted from the fuzzy set.

Example 3 (D10) (Dubois and Prade, 1980): Let U={positive real numbers}, be in reality an infinite set. Then, the fuzzy set A="real numbers close to 10", (see figure no. 1) may be defined as A={x, u_A, (x)}, when:

$$u_A(x)=1/\{1+[1/5(x-10)]^2\}$$

The basic concepts for fuzzy multiples - e.g. complement, support, normality, convexity, cardinality, etc. - have been developed by the founders of the theory of fuzzy sets of objects, such as Zadeh and Bellman, Dubois and Prade, Hwang and Yoon, and many others.

2. Basic research

The first attempt to put into practice the theory of fuzzy sets of objects for the analysis of multiple attribute decisions was made by Bellman and Zadeh (Bellman and Zadeh, 1970), who based their approach to decision making upon fuzzy sets of objects. Another approach was that of Zadeh (Zadeh, 1973), who outlined the possibility of using the „maxi-min" criterion with combine rational matrices. Papils (Papils, 1976) surveyed and modeled decisional matrixes by using a single item, i.e. a fuzzy set containing only one element (also Zadeh) (Zadeh, 1973). For this approach, the „maxi-min" rule is used to select the best alternative.

In 1978, Kickert (Kickert, 1979) summarized the application of the theory of fuzzy sets in problems regarding multiple attribute decisions, and Efstathiou (Efstathiou, 1979) performed a critical, essential overview on methods developed before 1979. Among them are the studies of Yager and Basson (Yager and Basson, 1975), Yager (Yager, 1982), Jain (Jain, 1976), Baldwin and Gild (Baldwin and Gild, 1979). Another exceptional summative study on the theory of fuzzy sets and its applications was made by Dubois and Prade (Dubois and Prade, 1982). They split up multiple attributes fuzzy sets into a fuzzy rating phase, in wich the fuzzy utility of each alternative is obtained and a second phase is reached, that of fuzzy ordering or making a fuzzy hierarchy (top), where fuzzy utilities are compared. In addition, the two researchers take into consideration both the fuzziness and the hazard, as possible fuzzy applications in decision analysis. Kaufman and Gupta (Kaufman and Gupta, 1985) also conceived an easy to read and easily comprehensible introduction to fuzzy arithmetic, a domain that is essential for fuzzy applications and algebraic calculus. One of the most comprehensive and up-to-date researches on fuzzy sets was made by Zimermann (Zimermann, 1985; Zimermann, 1987). This study, too deals with the problem of elaborating multiple attribute fuzzy decisions, as a 2-step process: the first achieves extraction or calculation of fuzzy utilities, which on the second level are compared by using for this purpose a fuzzy method of hierarchy. Zimermann's first book entitled, „Fuzzy Sets Theory and Applications" (Zimermann, 1985) lays more emphasis on the theory of fuzzy sets and its development rather than on applications. His second book, „Fuzzy Sets, Decision Making and Expert System" (Zimermann, 1987) is dedicated entirely to the process of elaborating fuzzy decisions and to expert systems. It represents one of the best scientific resources for studies regarding elaboration of fuzzy decisions.

No doubt, the best rounded and most fascinating work on modeling multiple attribute fuzzy decisions by using fuzzy sets is the book „Fuzzy Multiple Attribute Decision Making" (Chen and Hwang, 1992) by authors Shu-Jen Chen and Chin-Lai Hwang, with the cooperation of Frank P. Hwang, who offers the most comprehensive and thoroughly fundamented work in the field of fuzzy sets and which represents at the same time a bottomless fountain of information.

Apart from these works, we must show that there are also a few articles and studies regarding the analysis of fuzzy decisions in other books or collections of articles.

Table no. 1 presents the main themes of fuzzy sets the articles deal with.

Table 1

Books, monographs and studies

Themes approached	Authors	Year
• Basic theory of fuzzy sets and their applications	Zadeh	1979
	Bellman and Zadeh	1970
	Zadeh	1978
	Dinola and Venture	1986
	Dubois and Prade	1980
	Kandel	1986
	Kaufmann	1975
	Kaufmann and Gupta	1985
	Zimermann	1985
	Shu-Jen Chen; Chin-Lai Hwang	1992, 1989
• Theory of fuzzy sets and decision analysis	Gupta and Sanchez	1982; 1982
	Kaeprzyk and Yanger	1985
	Kaeprzyk and Orlovsky	1987
	Negoiță and Ralescu	1975
	Ionescu Gh. Gh.	1978
	Ionescu Gh. Gh.	1980, 1995
	Sanchez	1983, 1984
	Wang and Chang	1980
	Zimermann, Zadeh and Gaines (eds)	1984
	Zimermann	1987
		1984
• Theory of fuzzy sets and its general applications	Dubois and Prade	1988
	Gupta, Saridies and Gaines (eds)	1977

Themes approached	Authors	Year
	Gupta, Ragade and Yager (eds) Kaufman and Gupta (eds) Negoiță Zadeh, Fu, Tanaka and Shimura (eds)	1979 1988 1985, 1983 1975
• General decision analysis	Hwang and Masud Hwang and Yoon Hwang and Lin Kickert	1979 1981 1987 1978

3. Building the multiple criteria decision model by using fuzzy sets

Taking as starting point the usual formulation of a multiple criteria decision, we get:

$$A_i=|a_{ij}| \qquad i=\overline{1,m}; \qquad j=\overline{1,n}. \tag{5}$$

where:

a_{ij}= the consequence of alternative candidate A_i for criterion C_j, i.e. having to analyse the set of alternative candidates $\{A_i\}: i=\overline{1,m}$ according to the set of criteria $\{C_j\}: j=\overline{1,n}$, make the best choice according to the planned objectives.

This decisional situation can be solved by using the theory of fuzzy sets, with the help of which we can determine the affiliation of one alternative candidate A_i to the best variant.

Let us remember the definition of a fuzzy set as described by Zadeh; hence, for a usual, an ordinary set we can write a function of affiliation, which should take on values between 0 and 1, and respectively we deduce that:

$u_A(x) = 0$, if x does not belong to A: $x \notin A$
$u_A(x) \cong 0$, if x belongs a little to A: $x \in (<) A$
$u_A(x) = 0{,}5$, if x belongs fifty-fifty to A: $x \in (0{,}5) A$
$u_A(x) \cong 0$, if x belongs much to A: $x \in (>) A$
$u_A(x) = 1$, if x belongs completely to A: $x \in A$

In this way, the notion of fuzzy set can be extended to describe processes that happen in real life. This way, all phrases and expressions in ordinary speech, including adjectives, such as „bigger”, „smaller”, „small”, „significant”, „important”, „more precisely”, are fuzzy formulations.

The word „gray” epitomizes the most obviously the notion of fuzzy, as it represents a characteristic for a class of colors whose borders are not clear-cut, the extreme borderlines of non-affiliation have the their origin in two colors: white and black. Between these two limits of non-affiliation there is an infinity of fuzzy shades, as the passage from non-affiliation to affiliation is by no means abrupt, but gradual and richly shaded.

Every day we encounter fuzzy formulations in the entire specter of our daily lives. For example, the level of productivity of firm „A" is bigger than that of firm „B"; John is more handsome than Peter; University „X" is better than University „Z"; investment variant „I_i" is better than investment variant „I_j" ($i \neq j$; $i = \overline{1,m}$; $j = \overline{1,n}$).

Most decisional processes are fuzzy by nature because they are dependant on the context and on persons or groups of persons who make the decision. Ballman and Zadeh point out that, in most cases, the aims, restrictions and consequences of decisions are not well known.

Below is a model built by considering the characteristics of fuzzy sets, a model that allows us to eliminate the effects of imprecision and to adopt decisions with satisfactory accuracy.

Let A_i: $A_i=\{A_1, A_2, ..., A_i, ..., A_m\}$ be a set of alternatives of candidate actions, and C_j: $C_j=\{C_1, C_2, ..., C_j, ..., C_n\}$ a set of criteria used to evaluate the alternative candidate, i.e. the basic criteria for making the decision.

By noting A_1 the variant (alternative candidate) whose usefulness is 1, i.e. maximal, and A_0 the variant (alternative candidate) whose usefulness is 0, minimal, by using the properties of fuzzy sets we can determine the usefulness of the variants (alternative candidates) A_I , where: $A_i \neq A_1$ şi $A_i \neq A_0$.

Similarly, by noting:

a_{1j}= the most favorable consequence for criterion C_j;

a_{0j}= the most unfavorable consequence for criterion C_j;

a_{ij}= the consequence of a variant A_i for criterion C_j, where $i = \overline{1,m}$; $j = \overline{1,n}$.

Within a criterion C_j ($j = \overline{1,n}$) for a variant A_i ($i = \overline{1,m}$), where $A_i \neq A_1$ and $A_i \neq A_0$, we can establish a degree of closeness and one of distance for variant V_i from the optimal variant (the best variant). Taking into consideration the significances of candidate alternatives A_1 and A_0, we can conclude that, within a criterion C_j ($j = \overline{1,n}$), the degree of closeness plus the degree of distance for a candidate alternative A_i ($i = \overline{1,m}$ şi $A_i \neq A_1$ respectiv $A_i \neq A_0$) is 1, i.e.:

$$X_{ij} + \overline{X_{ij}} = 1 \qquad (6)$$

where:

X_{ij} = the degree of closeness of variant V_i in the context of criterion C_j vs. optimal variant (the best variant)

$\overline{X_{ij}}$ = the degree of distance of variant V_i in the context of criterion C_j vs. optimal variant (the best variant)

Establishing the degree of closeness and, respectively, of distance, varies according to the nature of the criterion. For criteria where $a_{ij} = \max_i \{a_{ij}\}$ for $i = \overline{1,m}$, i.e. the maximal consequence is the most favorable (with highest usefulness).

$$X_{ij} = \frac{a_{ij}}{a_{1j}}, \text{ for } i = \overline{1,m}\,;\ j = \overline{1,n} \quad (7)$$

and

$$\overline{X_{ij}} = 1\text{-}X_{ij} = \frac{a_{1j} - a_{ij}}{a_{1j}}, \text{ for } i = \overline{1,m}\,;\ j = \overline{1,n}\,. \quad (8)$$

We encounter this situation, for example, in the case of criteria of "maximal" respective profit; quality; volume of sales; production capacity; speed of work; productivity; etc.

In the case of criteria where $a_{ij} = \min\limits_{i} \{a_{ij}\}$ for $i = \overline{1,m}$, i.e. the minimum consequence is the most favorable (with highest usefulness)

$$X_{ij} = \frac{a_{1j}}{a_{ij}}, \text{ for } i = \overline{1,m}\,;\ j = \overline{1,n} \quad (9)$$

and

$$\overline{X_{ij}} = 1\text{-}X_{ij} = \frac{a_{ij} - a_{1j}}{a_{ij}}, \text{ for } i = \overline{1,m}\,;\ j = \overline{1,n}\,. \quad (10)$$

The minimal consequence is highest in the case of „minimum" criteria, such as: cost, purchase price, return duration, consumption (of labor force or other resources), fault rate (defects or wastes) etc.

Based on the degree of closeness, respectively, of distance, we can establish a dimension for the nature of utilities for each candidate alternative in the conditions of every criterion that can lead us to identifying the alternative candidate with the highest degree of belonging to the optimal variant.

The model for determining the degree of affiliation according to each criterion for every alternative candidate, which we note with Z_{ij} for $i = \overline{1,m}\,;\ j = \overline{1,n}$, has two variants, respectively, a first variant which uses function e^x, and a second variant which uses function e^{-x}.

a) Determining the degree of affiliation Z_{ij} with function e^x

In this case: $Z_{ij}=e^x$ (11)

where: $x = X_{ij} \cdot K_j$, for $i = \overline{1,m}\,;\ j = \overline{1,n}$.

Hence results that:

$$Z_{ij} = e^{X_{ij} \cdot K_j} : i = \overline{1,m}\,;\ j = \overline{1,n} \quad (12)$$

where:

Z_{ij} = the degree of affiliation of candidate alternative A_i to optimal (alternative) variant in conditions of criterion C_j;

K_j = the coefficient (or index) of importance of criterion C_j

X_{ij} = the degree of closeness of candidate alternative A_i vs. the optimal (alternative) variant according to criterion C_j

Obviously, $Z_{ij} \geq 1$, hence the degree of affiliation is not limited on the upper side.

b) Determining the degree of appurtenance z_{ij} with function e^{-x}

In this case: $Z_{ij}=e^{-x}$ (13)

where: $x = \overline{X}_{ij} \cdot K_j : i = \overline{1,m}\,; j = \overline{1,n}$.

results:

$$Z_{ij} = e^{-\overline{X}_{ij} \cdot K_j} : i = \overline{1,m}\,; j = \overline{1,n} \tag{14}$$

This variant presents the value of the degrees of appurtenance between 0 and 1, respectively:

$$0 \leq Z_{ij} \leq 1$$

which corresponds fully to the definition of fuzzy sets.

The ratio between the degrees of appurtenance of the same variant (alternative candidate), obtained by using the two methods, is as follows:

$$R_{ij} = \frac{e^{X_{ij}K_j}}{e^{-\overline{X}_{ij}K_j}} = e^{X_{ij}K_j + \overline{X}_{ij}K_j} = e^{K_j} \tag{15}$$

Consequently, the bigger / higher K_j, the bigger / higher R_{ij}.

Let us now analyze the ration between the degrees of affiliation of two variants (alternative candidates) A_i şi A_{i+1}, for each method:

a) – in the situation when we use function e^x

$$\frac{Z_{ij}}{Z_{i+1,j}} = \frac{e^{X_{ij}K_j}}{e^{X_{i+1,j}K_j}} = e^{K_j(X_{ij} + X_{i+1,j})} \tag{16}$$

The bigger the importance value of the criterion and the difference between the degrees of closeness between the two variants (alternatives A_i and A_{i+1}) vs. the optimal variant (alternative), the bigger this ration will be.

If:

$$X_{ij} = X_{i+1,j} \Rightarrow \frac{Z_{ij}}{Z_{i+1,j}} = 1; \tag{17}$$

$$X_{ij} > X_{i+1,j} \Rightarrow \frac{Z_{ij}}{Z_{i+1,j}} > 1; \tag{18}$$

$$X_{ij} < X_{i+1,j} \Rightarrow \frac{Z_{ij}}{Z_{i+1,j}} < 1 \qquad (19)$$

b) – in the case of using function e^{-x}

$$\frac{Z_{ij}}{Z_{i+1,j}} = \frac{e^{-\overline{X}_{ij}K_j}}{e^{-\overline{X}_{i+1,j}K_j}} = e^{-K_j(\overline{X}_{ij}+\overline{X}_{i+1,j})} \qquad (20)$$

We can therefore conclude that the ration between the degrees of affiliation of two variant (alternative) candidates is the same, whether we calculate it by function e^x or e^{-x}.

By using the two functions e^x and e^{-x}, we obtain the degrees of affiliation which reflect simultaneously both the level of consequence of the variants (by way of degrees of closeness or distance vs. the optimal variant), and the importance of the criteria.

This way, we have a real, correct solution, because primarily there is no direct proportionality between consequences and the corresponding degrees of affiliation, which is quite close to reality and encountered fairly often in actual decisional situation. Secondly, the degree of objectivity in establishing degrees of affiliation increases, since subjective probabilities are kept out because both the level of the consequences and the importance of the criteria are considered.

With the help of the matrix of degrees of affiliation we can make use of any decisional criterion of the theory of decision, respectively, Wald's optimistic criterion, Baumall's super-optimistic criterion, Savage's criterion of regrets, the Bayes-Laplace criterion or the additive model - as we shall illustrate our opinions later on.

Estimating degrees of affiliation with the help of functions e^x and e^{-x}, raises an additional problem, namely, establishing the importance value for the evaluation criteria.

The making of a hierarchy of the criteria by giving importance value must be based on an analysis of certain factors for each concrete given situation. We deem the following to be essential:

- the nature of variants (alternative candidates) to be studies;
- the specifics of the decisional process;
- the nature of the evaluation and selection criteria;
- the decision maker's or organization's objectives;
- the organizational synergy;
- etc.

The importance values given to the criteria must be result from a collective or group decision, in the sense that it must involve as many as possible specialists from various organizational positions or field, respectively: economic, technological, finances-accounting, research, development, marketing, sociology, psychology, management of human resources etc., obviously according to the existing decisional context.

Determining the importance values of the criteria can be achieved by several methods according to the model of the decision making, of accomplishing the option. In our case, respectively, the model built by using fuzzy functions, we shall consider the criteria C_1 ... C_j ... C_n and at the same time the persons P_1 ... P_k ... P_s selected for making a hierarchy of the criteria.

To make the hierarchy, we shall request that every person should give a grade for each criterion.

By noting with n_{kj} the grade given by person P_k to criterion C_j, we obtain the following matrix (see table no. 2)

Table 2

Grades Matrix

P_k \ C_j	C_1	....	C_j	....	C_n
P_1	n_{11}	....	n_{1j}	....	n_{1n}
⋮	⋮		⋮		⋮
P_k	n_{k1}	....	n_{kj}	....	n_{kn}
⋮	⋮		⋮		⋮
P_s	n_{s1}	....	n_{sj}	....	n_{sn}
	N_1	....	N_j	....	N_n

If we wish to make it possible for one person to make a strict hierarchy of the criteria, the grades to be given must range from 1 to no more than the number of criteria, i.e.

$$1 \le n_{kj} \le N \text{, for } k = \overline{1,s};\ j = \overline{1,n} \tag{21}$$

where N is the maximum grade that can be given to one criterion.

The sum of the grades obtained by one criterion will be:

$$N_j = \sum_{k=1}^{s} n_{kj} \text{, for } j = \overline{1,n} \tag{22}$$

The maximum number of points that can be obtained by one criterion is $(N_j)max=s\cdot N$, and the minimal number of points is $(N_j)min=s\cdot 1=s$.

On the basis of these elements, we can calculate the importance criterion for each criterion in the following ways:

$$K_j = \frac{N_j}{(N_j)\min} = \frac{N_j}{s} \text{, and } 1 \le K_j \le N;\ j = \overline{1,n} \qquad \text{(a)}$$

$$K_j = \frac{N_j}{(N_j)\max} = \frac{N_j}{s \cdot N} \text{, and } \frac{1}{N} \le K_j \le 1;\ j = \overline{1,n} \qquad \text{(b)} \tag{23}$$

$$K_j = \frac{(N_j)\min}{N_j} = \frac{s}{N_j}, \text{ and } \frac{1}{N} < K_j < 1;\ j = \overline{1,n} \qquad \text{(c)}$$

$$K_j = \frac{(N_j)\max}{N_j} = \frac{N \cdot s}{N_j}, \text{ and } 1 < K_j < N;\ j = \overline{1,n} \qquad \text{(d)}$$

The relations above lead us to several interesting conclusions:

- In all the cases we obtain strictly positive importance values.
- The top limit of the importance values is N.
- All four calculus relations are applicable both in the case of the affiliation function $Z_{ij} = e^{X_{ij} \cdot K_j}$ and in that of the affiliation function $Z_{ij} = e^{-\overline{X}_{ij} \cdot K_j}$, but in a different way, namely, according to the way of giving the grades.

(1) if the grades are directly proportional with the degree of importance given to the criteria, respectively, if a criterion considered important is given a grade that is higher than that given to one considered less important, and vice versa, then an analysis of the variation of the functions $Z_{ij} = e^{X_{ij} \cdot K_j}$ or $Z_{ij} = e^{-\overline{X}_{ij} \cdot K_j}$ shows that, to determine importance value, calculus relations (a) or (b) must be used for the former function, while for the latter function we must use relations (c) or (d).

(2) if the grades given are inversely proportional with the degree of importance conferred to the criteria (low grades are given to important criteria), relations (c) or (d) will be used for function $Z_{ij} = e^{X_{ij} \cdot K_j}$, while relations (a) or (b) are used for function $Z_{ij} = e^{-\overline{X}_{ij} \cdot K_j}$.

Estimation of the degrees of affiliation according to function $Z_{ij} = e^{X_{ij} \cdot K_j}$ involves: the greater the importance of the criterion, the higher must be the importance value given. Conversely, when using affiliation function $Z_{ij} = e^{-\overline{X}_{ij} \cdot K_j}$, more important criteria will be given lower importance values.

Although, theoretically, in all four calculus relations of importance values there is a highest and a lowest limit, for practical considerations, certain calculus relations are preferred for function $Z_{ij} = e^{X_{ij} \cdot K_j}$, and other relations for function $Z_{ij} = e^{-\overline{X}_{ij} \cdot K_j}$.

Function e^x has a range of values between 1 and ∞. Considering that the use of this function to establish degrees of appurtenance is advisable for the interval 1 and 2, we get:

$$(Z_{ij})\min = (e^{X_{ij}\cdot K_j})\min = 1$$

$$(Z_{ij})\max = (e^{X_{ij}\cdot K_j})\max = 2$$

From relation $(Z_{ij})\min = (e^{X_{ij}\cdot K_j})\min = 1$, it results that $X_{ij}\cdot K_j = 0$, because K_j>0, (X_{ij})min= 0. Consequently, $Z_{ij} = 1$ when $X_{ij} = 0$.

From relation $(Z_{ij})\max = (e^{X_{ij}\cdot K_j})\max = 2$, it results that $X_{ij}\cdot K_j = 0.7$ and consequently $K_j = \frac{0.7}{(X_{ij})\max}$; (X_{ij})max= 1, resulting that $(K_j)\max = 0.7$

The facts presented above lead us to the conclusion that while using the function $Z_{ij} = e^{X_{ij}\cdot K_j}$, $0 \le K_j \le 0.7$, the importance values must be calculated by using relations (b) or (c), obviously, according to the way of giving the grades.

Relations (b) and (c) show that K_{ij} can reach the value 1, for which $(Z_{ij})\max = (e^{X_{ij}\cdot K_j})\max = e^1 = 2.71828$, which can be admitted in a certain way. At the same time, it is highly unlikely that we will obtain, for a certain criterion, an importance equal with 1. This would, in fact, mean that absolutely all persons P_1 ... P_s consider unanimously the respective criterion to be the most important, an essentially particular, and uninteresting, fact.

Hence the idea that we wish to highlight is that, when using function $Z_{ij} = e^{X_{ij}\cdot K_j}$, the importance values given to the criteria must be sub-unitary, even if they will not always be smaller than 0.7.

If we use supra-unitary values, the degrees of affiliation obtained will be highly dispersed, e.g. for K_j=3 and X_{ij}=1 (for every criterion, for the most favorable consequence X_{ij}=1), we obtain $Z_{ij}=e^3=20.086$.

Let us analyze the case of function e^{-x}: here values vary between 1 and 0, which corresponds to the definition of the fuzzy set. For x=0, i.e. $\overline{X}_{ij}\cdot K_j = 0$, $e^{-0}=1$, consequently $(Z_{ij})\max = (e^{-\overline{X}_{ij}\cdot K_j})\max = 1$, for $\overline{X}_{ij}\cdot K_j = 0$. Since K_j>0, it results that Z_{ij}=1 for $\overline{X}_{ij} = 0$.

For x=9.9, respectively $\overline{X}_{ij}\cdot K_j = 9.9$: $Z_{ij}=e^{-9.9}=0.0050$ i.e. very close to 0. We can consider (Z_{ij})min for $(X_{ij}\cdot K_j)$max=9.9; $(X_{ij}\cdot K_j)$max=1, so that $(K_j)\max = \frac{9.9}{1} \approx 10$. Therefore, in order to use the entire interval of variation between 0 and 1 of the function e^{-x}, K_j must take values no higher than 10. Supra-unitary values for K_j are obtained by relations (a) and (d), these being preferable when calculating degrees of appurtenance according to this function. Although we obtain supra-unitary importance values for all criteria, this does not alter the essence of the problem, because for a K_j=1, even if $\overline{X}_{ij} = 1$ (i.e. the case of the most unfavorable variant, but most important criterion), we obtain Z_{ij}=e^{-1}=0.368788, while for a K_j=5, for example (i.e. the least important criterion) and $\overline{X}_{ij} = 1$, we obtain Z_{ij}=e^{-5}=0.00674, therefore, a much higher degree of appurtenance.

The objection can be raised that we have limited N, and thus also the number of evaluation criteria n to 10.

In practical situations, anyway, it is not advisable to use an exaggerated number of criteria because, this way, we would „dilute" or „smash up" the objectives targeted, or we would in fact „replace" certain criteria. Similarly, it is highly unlikely that we should obtain K_j=10, even when using relations (a) or (b), because that would mean „unanimity" for all those who give the grades concerning the most important criterion - which, as we have already shown, is not an interesting situation. If, however, we obtain an importance value higher than 10, we can reduce it to 10, by reducing also proportionally the values for the other criteria. For example, we obtain K_1=4; K_2=6; K_3=10; K_4=16. In this case, we correct each value obtained by the ratio 10/16 and we obtain:

K_1=4·10/14=2.5; K_2=6·10/16=3.75; K_3=10·10/16=6.25; K_4=16·10/16=10.

Therefore, if (K_j)max=N>10, we correct each value with the ratio 10/N.

In conclusion, when using the function $Z_{ij} = e^{X_{ij} \cdot K_j}$, it is advisable to attribute importance values to criteria calculated by using the relations (b) and (c), with $0 \le K_j \le 0{,}7\ (j = \overline{1,n})$; conversely, when using the appurtenance function $Z_{ij} = e^{-\overline{X}_{ij} \cdot K_j}$, it is advisable to calculate importance values by using the relations (a) or (d), with $1 \le K_j \le 10\ (j = \overline{1,n})$.

Estimating degrees of appurtenance by using one of the functions described above is possible, and at the same time it offers advantages in the case of criteria where consequences are expressed qualitatively, not quantitatively.

For example, for the criterion "quality", expressed by a number of levels of appreciation, an odd number of levels of appreciation is recommended, so that there should be a point of balance. For the above criterion, let us suppose that that we have the following levels: very good, good, satisfactory, poor, very poor.

For the extreme levels and for the point of balance, we can establish the degrees of closeness or distance objectively, namely:

For the extreme steps and the point of equilibrium we can determine the degree of distance or closeness in a objective way, respectively:

Level	X_{ij}	$\overline{X}_{ij}$
Very good	1	0
Satisfactory	0,5	0,5
Very poor	0	1

For quality „good” we can give, for example, X_{ij} = 0.75 and, respectively, $\overline{X}_{ij}$ = 0.25, and for „poop” X_{ij} = 0.25 and, respectively, $\overline{X}_{ij}$ = 0.75. After establishing the importance value for the criterion „quality”, we can calculate the degrees of affiliation by using one of the functions proposed. In this way, „subjectivity” is reduced considerably with respect to the situation where we use the calculus method of linear interpolation.

4. Numerical Example (Hwang and Yoon, 1981)

A country decided to purchase a fleet of jet fighters from the US. The Pentagon officials offered the typical information for four models that could be sold to that country. The team of Air Force analysts (10 persons) for that country agreed that six characteristics (attributes or criteria) should be considered. They are: maximum speed (C_1), ferry range (C_2), maximum payload (C_3), purchasing cost (C_4), reliability (C_5) and ease of handling (C_6). The measurement units for these attributes are: match, miles, pounds, dollars (in millions), high-low scale and high-low scale, respectively. The decision matrix for the fighter aircraft selection problems is presented in table no. 3.

Table 3

The decision matrix

A_i \ C_j	C_1	C_2	C_3	C_4	C_5	C_6
A_1	2,0	1500	20.000	5,5	average	very high
A_2	2,5	2700	18.000	6,5	low	average
A_3	1,8	2000	21.000	4,5	high	high
A_4	2,2	1800	20.000	5,0	average	average

First we shall make a hierarchy of the criteria - for which we considered that the team of the purchasing country consists of 10 persons, who made a hierarchy as presented in table no. 4

Table 4

The criteria hierarchy

P_k \ C_j	C_1 (max)	C_2 (max)	C_3 (max)	C_4 (min)	C_5 (max)	C_6 (max)
P_1	1	6	3	2	4	5
P_2	2	4	6	3	1	5
P_3	3	5	4	6	2	1
P_4	6	2	1	4	3	5
P_5	5	6	3	4	1	2
P_6	2	5	1	6	3	4
P_7	4	5	6	3	1	2
P_8	1	5	6	4	2	3
P_9	2	1	3	4	6	5
P_{10}	3	4	2	5	1	6
N_j	29	43	35	41	24	38
(b) $K_j = \frac{N_j}{N \cdot s}$	0,48	0,72	0,58	0,68	0,40	0,63
(d) $K_j = \frac{N \cdot s}{N_j}$	2,06	1,40	1,71	1,46	2,5	1,58

Note: We use formula (b) for K_j to calculate degrees of affiliation „Z_{ij}" with function e^x

– we use formula (d) for K_j to calculate degrees of affiliation „Z_{ij}" with function e^{-x}.

Accordingly, we shall first survey the use of function e^x for calculating degrees of appurtenance. To do so, we first calculate the matrix of degrees of appreciation (see table no. 5)

Then we calculate the matrix of appurtenance degrees x, where $x = X_{ij \cdot K_j}$ (see table no. 6)

Subsequently, we determined the matrix of appurtenance degrees according to function e^x, as shown in table no. 7

Table 5

Matrix of closeness degrees (X_{ij})

A_i \ C_j	C_1 (max)	C_2 (max)	C_3 (max)	C_4 (min)	C_5 (max)	C_6 (max)
A_1	0,80	0,55	0,95	0,81	0,50	1,00
A_2	1,00	1,00	0,86	0,69	0,25	0,50
A_3	0,72	0,74	1,00	1,00	0,75	0,75
A_4	0,88	0,66	0,95	0,90	0,50	0,50
K_j	0,48	0,72	0,58	0,68	0,40	0,63

$$X_{11} = \frac{2.0}{2.5} = 0.8 \qquad X_{12} = \frac{1500}{2700} = 0.55$$

$$X_{21} = \frac{2.5}{2.5} = 1 \qquad X_{22} = \frac{2700}{2700} = 1$$

$$X_{31} = \frac{1.8}{2.5} = 0.72 \qquad X_{32} = \frac{2000}{2700} = 0.74$$

$$X_{41} = \frac{2.2}{2.5} = 0.88 \qquad X_{42} = \frac{1800}{2700} = 0.66$$

$$X_{13} = \frac{20000}{21000} = 0.95 \qquad X_{14} = \frac{4.5}{5.5} = 0.81$$

$$X_{23} = \frac{18000}{21000} = 0.86 \qquad X_{24} = \frac{4.5}{6.5} = 0.69$$

$$X_{33} = \frac{21000}{21000} = 1.00 \qquad X_{34} = \frac{4.5}{4.5} = 1.00$$

$$X_{43} = \frac{20000}{21000} = 0.95 \qquad X_{44} = \frac{4.5}{5.0} = 0.9\ X_{ij}$$

For non-quantitative criteria we use the evaluation scale (see figure no. 3 and figure no. 4)

	Very low	Low	Medium	High	Very high
X_{ij}	0	0.25	0.50	0.75	1.00

Figure 3. Scale for degree of closeness

	Very low	Low	Medium	High	Very high
$\overline{X}_{ij}$	1.00	0.75	0.5	0.25	0

Figure 4. Scale for degree of distance

Scale for degree of distance

Table 6

Matrix : ($x=X_{ij}\cdot K_j$)

A_i \ C_j	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.3840	0.3960	0.5510	0.5508	0.20	0.6300
A_2	0.4800	0.7200	0.4988	0.4692	0.10	0.3150
A_3	0.3456	0.5328	0.5800	0.5800	0.30	0.4725
A_4	0.4224	0.4752	0.5510	0.6120	0.20	0.3150

Table 7

Matrix of degrees of affiliation calculated by using function e^x ($Z_{ij}=e^x=e^{X_{ij}\cdot K_j}$)

A_i \ C_j	C_1	C_2	C_3	C_4	C_5	C_6	$\sum_{j=1}^{6} Z_{ij}$
A_1	1.46228	1.49182	1.73325	1.73325	1.22140	1.87761	9.51961
A_2	1.61607	2.05443	1.64872	1.59999	1.10517	1.37713	9.40151
A_3	1.41907	1.69893	1.78604	1.97388	1.34986	1.59999	9.82777
A_4	1.52196	1.61607	1.73325	1.84043	1.22140	1.37713	9.31024

Consequently, by applying the additive model, respectively, the optimal decision is given by the alternative candidate for which the sum of degrees of affiliation for each criterion is maximum, i.e.:

$$D^0=(G+)=\max_i\left[\sum_{j=1}^{6} Z_{ij}\right]=\max_i[9.51961; 9.40151; 9.82777; 9.31024]=9.82777 \Rightarrow A_3$$

the best alternative candidate, and the hierarchy for favorite alternative candidates will be:

I (9.82777) $\Rightarrow A_3$
II (9.51961) $\Rightarrow A_1$
III (9.40151) $\Rightarrow A_2$
IV (9.31024) $\Rightarrow A_4$

Thus: $A_3\,\mathscr{P}\,\{A_1\, A_2\, A_4\}$ where $\mathscr{P}$ = preferenced or prefered
$A_1\,\mathscr{P}\,\{A_2\, A_4\}$
$A_2\,\mathscr{P}\,\{A_4\}$
$A_4\,\mathscr{P}\,\{\emptyset\}$

We notice that distancing is obvious, but not sudden of disproportionate by in terms o appurtenance and non-affiliation; and that affiliation is very finely nuanced.

Let us now illustrate the model of affiliation with function e^{-x}, respectively $e^{-X_{ij} \cdot K_j}$, which, in our opinion, is much more exact, as the limits of affiliation are 0 and 1, which corresponds to the definition of the fuzzy set.

Let us solve the decisional problem by using degrees of affiliation calculated with function e^{-x}. According to the methodology, we first calculate the matrix of for degrees of distance $\overline{X}_{ij}$ (see table no. 8). Then we calculate matrix: $\overline{X}_{ij} \cdot K_j$ (see table no. 9). Eventually, we calculate the matrix for degrees of appurtenance (see table no. 10).

Table 8

Matrix for degrees of distance ($\overline{X}_{ij}$)

A_i \ C_j	C_1 (max)	C_2 (max)	C_3 (max)	C_4 (min)	C_5 (max)	C_6 (max)
A_1	0.20	0.45	0.05	0.19	0.50	0.00
A_2	0.00	0.00	0.14	0.31	0.75	0.50
A_3	0.28	0.26	0.00	0.00	0.25	0.25
A_4	0.12	0.34	0.005	0.10	0.50	0.50
K_j	2.06	1.40	1.71	1.46	2.5	1.58

Table 9

Matrix: ($x = \overline{X}_{ij} \cdot K_j$)

A_i \ C_j	C_1 (max)	C_2 (max)	C_3 (max)	C_4 (min)	C_5 (max)	C_6 (max)
A_1	0.412	0.63	0.0855	0.2774	1.25	0.00
A_2	0.0	0.00	0.2394	0.4526	1.875	0.79
A_3	0.5768	0.364	0.00	0.00	0.625	0.395
A_4	0.2472	0.476	0.0855	0.146	1.25	0.79

Table 10

Matrix of the affiliation degrees calculated by formula e^{-x} ($x = \overline{X}_{ij} \cdot K_j$)

A_i \ C_j	C_1	C_2	C_3	C_4	C_5	C_6	$\sum_{j=1}^{6} Z_{ij}$
A_1	0.66365	0.53259	0.91393	0.75578	0.28650	1.0	4.15245
A_2	1.0	1.0	0.78663	0.63763	0.15260	0.45384	4.03070
A_3	0.55990	0.69768	1.0	1.0	0.53259	0.67032	4.46049
A_4	0.77880	0.61878	0.91393	0.86071	0.28650	0.45384	3.91256

By applying the additive model, respectively, the optimal decision is given by the alternative candidate for which the sum of affiliation degrees for each criterion is maximum, i.e.:

$$D^0 = (G+) = \max_i \left[\sum_{j=1}^{6} Z_{ij} \right] = \max_i [4.15245; 4.03070; 4.46049; 3.91256] = 4.46049 \Rightarrow A_3$$

the best alternative candidate, and the hierarchy according to the order of preference of alternative candidates will be:

I (4.46049) $\Rightarrow A_3$
II (4.15245) $\Rightarrow A_1$
III (4.03070) $\Rightarrow A_2$
IV (3.91256) $\Rightarrow A_4$

Thus: $A_3 \mathcal{P} \{A_1 A_2 A_4\}$
$A_1 \mathcal{P} \{A_2 A_4\}$
$A_2 \mathcal{P} \{A_4\}$
$A_4 \mathcal{P} \{\emptyset\}$

We notice that we are dealing with an extremely fine, but crystal clear categorization on an extremely restricted interval, i.e. between 3.91256 and 4.46049. Similarly, we notice that we obtain the same classification as the one ontained when using function e^x for calculating degrees of appurtenance, but we think that function e^{-x} is much more exact, as it has a very precise interval for the degree of appurtenance, i.e. between 0 and 1, respectively between non-appurtenance and total appurtenance, which corresponds to the definition of the fuzzy set. We recommend that function e^{-x} be used for calculating degrees of appurtenance.

Obviously, for to make decisions apart from the additive model (G+) we can use any of the five rules of decision. To highlight this idea, we will apply them for the two relations of calculus of Z_{ij}, with function e^x and, respectively, with function e^{-x}.

1) *The optimistic criterion* (maximax): By analyzing the matrix of degrees of affiliation, determined by using function e^x (table no. 7), we obtain the vector column:

$$D_0 = \max_j \begin{bmatrix} 1.87761 \\ 2.05443 \\ 1.97388 \\ 1.84073 \end{bmatrix} = \max_i [2.05443] \Rightarrow A_2$$

and the scale will be:

I = 2.05443 $\Rightarrow A_2$
II = 1.97388 $\Rightarrow A_3$
III = 1.87761 $\Rightarrow A_1$
IV = 1.84043 $\Rightarrow A_4$

Thus: $A_2 \mathcal{P} \{A_1 A_3 A_4\}$

$A_3 \mathscr{P} \{A_1\ A_4\}$
$A_1 \mathscr{P} \{A_4\}$
$A_4 \mathscr{P} \{\varnothing\}$

2) *The pessimistic criterion* (maxmini): By analyzing the matrix of affiliation degrees determined by using function e^x (see table no. 7), we obtain the vector column:

$$D_0 = \min_j \begin{bmatrix} 1.22140 \\ 1.10517 \\ 1.34986 \\ 1.22140 \end{bmatrix} = \max_i [1.34986] \Rightarrow A_3$$

and the scale will be:

I = 1.34986 $\Rightarrow A_3$
II = 1.22140 $\Rightarrow \{A_1;\ A_4\}$
III = 1.10517 $\Rightarrow \{A_2\}$
IV = 1.84043 $\Rightarrow A_4$

Thus: $A_3 \mathscr{P} \{A_1\ A_2\ A_4\}$
$A_1 \mathscr{P} \{A_2\}$
$A_4 \mathscr{P} \{A_2\}$
$A_2 \mathscr{P} \{\varnothing\}$

3) *The criterion of regrets* (Savage). Based on the matrix of affiliation degrees, we calculate the matrix of regrets (see table no. 11). The decision is of the minimax type, hence results the vector:

$$D_0 = \max_j \begin{bmatrix} 0.56261 \\ 0.50048 \\ 0.55550 \\ 0.52790 \end{bmatrix} = \min_i [0.50048] \Rightarrow A_2$$

The top of hierarchies will be:

I = 0.50048 $\Rightarrow A_2$
II = 0.52790 $\Rightarrow A_4$
III = 0.55550 $\Rightarrow A_3$
IV = 0.56261 $\Rightarrow A_1$

Thus: $A_2 \mathscr{P} \{A_1\ A_3\ A_4\}$
$A_4 \mathscr{P} \{A_1\ A_3\}$
$A_3 \mathscr{P} \{A_1\}$
$A_1 \mathscr{P} \{\varnothing\}$

If we apply the additive model to the criterion of regrets, we obtain the vector (see table no. 11)

$$\left[\sum_{j=1}^{6} r_{ij}\right] \Rightarrow D_0 = \min_i \left[\sum_{j=1}^{6} r_{ij}\right] = \min_i \begin{bmatrix} 1.13828 \\ 1.25638 \\ 1.03012 \\ 1.81829 \end{bmatrix} \Rightarrow [1.03012] \Rightarrow A_3$$

and the top of hierarchies will be:

I = 1.03012 $\Rightarrow A_3$
II = 1.13828 $\Rightarrow A_1$
III = 1.25638 $\Rightarrow A_2$
IV = 1.81829 $\Rightarrow A_4$

Thus: $A_3 \mathcal{P} \{A_1\ A_2\ A_4\}$
$A_1 \mathcal{P} \{A_2\ A_4\}$
$A_2 \mathcal{P} \{A_4\}$
$A_4 \mathcal{P} \{A_2\}$

We notice that the same result is obtained as in the case of the additive model, where we used degrees of appurtenance.

Table 11

Matrix of regrets (r_{ij})

A_i \ C_j	C_1	C_2	C_3	C_4	C_5	C_6	$\sum_{j=1}^{6} r_{ij}$
A_1	0.15379	0.56261	0.05279	0.24063	0.12846	0.0	1.13828
A_2	0.0	0.0	0.13732	0.37389	0.24469	0.50048	1.25638
A_3	0.19700	0.55550	0.0	0.0	0.0	0.27762	1.03012
A_4	0.08964	0.43836	0.52790	0.13345	0.12846	0.50048	1.81829

4) *The criterion of realism* (Hurvicz). Let us consider $\alpha = 0.6$ then $(1-\alpha) = 0.4$; with these values, we calculate the values of expectancy of payments (degrees of appurtenance). Thus:

$VE_1 = 0.6\times1.87761 + 0.4\times1.22140 = 1.126566 + 0.48856 = 1.615126$
$VE_2 = 0.6\times2.05443 + 0.4\times1.10517 = 1.232658 + 0.442068 = 1.674726$
$VE_3 = 1.97388\times0.6 + 0.4\times1.10517 = 1.18438 + 0.539944 = 1.724324$
$VE_4 = 0.6\times1.84043 + 0.4\times1.22140 = 1.104258 + 0.48856 = 1.592818$

$$D_0 = \max_i[VE_i] = \max_i \begin{bmatrix} 1.615126 \\ 1.674726 \\ 1.724324 \\ 1.592818 \end{bmatrix} = [1.724324] \Rightarrow A_3$$

and the top of hierarchies will be:

I = 1.724324 $\Rightarrow A_3$
II = 1.674726 $\Rightarrow A_2$
III = 1.615126 $\Rightarrow A_1$
IV = 1.592818 $\Rightarrow A_4$

Thus: $A_3 \mathcal{P} \{A_1\ A_2\ A_4\}$
$A_2 \mathcal{P} \{A_1\ A_4\}$
$A_1 \mathcal{P} \{A_4\}$
$A_4 \mathcal{P} \{\varnothing\}$

5) *The „Bayes-Laplace" criterion***.** This criterion suggests that the hierarchy is established according to the relation $\max_i[VE_i]$, where:

$VE_i = \frac{1}{n}\sum_{j=1}^{n} z_{ij}$, where „n" represents the number of criteria, so that we have

$$VE_1 = \frac{1}{6} \cdot 9.51961 = 1.5866016$$

$$VE_2 = \frac{1}{6} \cdot 9.40151 = 1.5669183$$

$$VE_3 = \frac{1}{6} \cdot 9.82777 = 1.6379616$$

$$VE_4 = \frac{1}{6} \cdot 9.31024 = 1.5517066$$

$$D_0 = \max_i[VE_i] = \max_i \begin{bmatrix} 1.5866016 \\ 1.5669183 \\ 1.6379616 \\ 1.5517066 \end{bmatrix} = [1.6379616] \Rightarrow A_3$$

And the hierarchy top will be:

I = 1.6379616 $\Rightarrow A_3$
II = 1.5866016 $\Rightarrow A_1$
III = 1.5669183 $\Rightarrow A_2$
IV = 1.5518066 $\Rightarrow A_4$

Thus: $A_3 \mathcal{P} \{A_1 A_2 A_4\}$
$A_1 \mathcal{P} \{A_2 A_4\}$
$A_1 \mathcal{P} \{A_4\}$
$A_4 \mathcal{P} \{\emptyset\}$

We have obtained the same result as in the case of the additive model of appurtenance degrees, which is only logical.

We proceed in the same way to establish the situation of degrees of affiliation calculated by using function e^{-x}.

1) *The optimistic criterion.* Analyzing the matrix of affiliation degrees (see table no. 10), we obtain the column vector:

$$D_0 = \max_j \begin{bmatrix} 1.0 \\ 1.0 \\ 1.0 \\ 0.91393 \end{bmatrix} = \max_i [1.0] \Rightarrow \{A_1; A_2; A_3\}$$

which shows a state of indeterminacy.

2) *The pessimistic criterion.* An analysis of the degrees of affiliation (see table no. 10) leads us to the column vector:

$$D_0 = \min_j \begin{bmatrix} 0.28650 \\ 0.15260 \\ 0.53259 \\ 0.28650 \end{bmatrix} = \max_i [0.53259] \Rightarrow A_3$$

And the hierarchy top will be:
I = 0.53259 $\Rightarrow A_3$
II = 0.28650 $\Rightarrow A_1$
III = 0.15260 $\Rightarrow \{A_2; A_4\}$

Thus: $A_3 \mathcal{P} \{A_1 A_2 A_4\}$
$A_1 \mathcal{P} \{A_2 A_4\}$
$\{A_2 A_4\} \mathcal{P} \{\emptyset\}$

Here again we have a certain indeterminacy.

Table 12

Matrix of regrets (r_{ij})

C_j / A_i	C_1	C_2	C_3	C_4	C_5	C_6	$\sum_{j=1}^{6} r_{ij}$
A_1	0.33635	0.46741	0.08907	0.24428	0.24609	0.0	1.38320
A_2	0.0	0.0	0.21337	0.36237	0.37999	0.54616	1.50189
A_3	0.4401	0.30232	0.0	0.0	0.0	0.32968	1.07110
A_4	0.2212	0.38122	0.08907	0.13929	0.24609	0.54616	1.62303

If we use the additive model for regrets (see table no. 12 $\sum_{j=1}^{n} r_{ij}$), we obtain a hierarchy with the help of the relation $\min_i \left[\sum_{j=1}^{n} r_{ij} \right]$, so that

I = 1.07110 $\Rightarrow A_3$
II = 1.38320 $\Rightarrow A_1$
III = 1.50189 $\Rightarrow A_2$
IV = 1.62303 $\Rightarrow A_4$

Thus: $A_3 \mathscr{P} \{A_1\, A_2\, A_4\}$
$A_1 \mathscr{P} \{A_2\, A_4\}$
$A_2 \mathscr{P} \{A_4\}$
$A_4 \mathscr{P} \{\emptyset\}$

The solution is the same with this of the additive model, and it is logically.

3) *The criterion of realism* (Hurwicz). We take $\alpha = 0.6$, then $(1-\alpha) = 0.4$. Consequently, the values of expectancy for the payments (the degrees of affiliation, Z_{ij}) for each alternative candidate is:

VE_1=0.6·1.0+0.4·0.28650=0.6+0.1146=0.7146
VE_2=0.6·1.0+0.4·0.1526=0.6+0.06104=0.66104
VE_3=0.6·1.0+0.4·0.533259=0.6+0.213036=0.813036
VE_4=0.6·0.91393+0.4·0.28650=0.548358+0.1146=0.662958

results that:

$$D_0 = \max_i [VE_i] = \max_i \begin{bmatrix} 0.7146 \\ 0.66104 \\ 0.813036 \\ 0.662958 \end{bmatrix} = [0.813036] \Rightarrow A_3$$

and the full hierarchy will be:

I = 0.813036 $\Rightarrow A_3$

II = 0.7146 $\Rightarrow A_1$
III = 0.662958 $\Rightarrow A_4$
IV = 0.66104 $\Rightarrow A_2$

Thus: A_3 $\mathscr{P}$ $\{A_1\ A_2\ A_4\}$
A_1 $\mathscr{P}$ $\{A_2\ A_4\}$
A_4 $\mathscr{P}$ $\{A_2\}$
A_2 $\mathscr{P}$ $\{\emptyset\}$

4) *The „Bayes-Laplace" criterion*. By applying this criterion results the vector:

$$VE_1 = \frac{1}{6} \cdot 4.15245 = 0.692075$$

$$VE_2 = \frac{1}{6} \cdot 4.03070 = 0.6717833$$

$$VE_3 = \frac{1}{6} \cdot 4.46049 = 0.743415$$

$$VE_4 = \frac{1}{6} \cdot 3.91256 = 0.6520933$$

$$D_0 = \max_i [VE_i] = \max_i \begin{bmatrix} 0.692075 \\ 0.6717833 \\ 0.743415 \\ 0.6520933 \end{bmatrix} = [0.743415] \Rightarrow A_3$$

and the hierarchy is:

I = 0.743415 $\Rightarrow A_3$
II = 0.692075 $\Rightarrow A_1$
III = 0.6717833 $\Rightarrow A_2$
IV = 0.6520933 $\Rightarrow A_4$

Thus: A_3 $\mathscr{P}$ $\{A_1\ A_2\ A_4\}$
A_1 $\mathscr{P}$ $\{A_2\ A_4\}$
A_2 $\mathscr{P}$ $\{A_4\}$
A_4 $\mathscr{P}$ $\{\emptyset\}$

We notice, logically, that the decision is identical with that for the additive model using degrees of appurtenance „z_{ij}".

The overall conclusion is that no matter what variant we might use for calculating degrees of appurtenance, we employ either function e^x of function e^{-x}, the decision that results is the same. We must underline, however, that the second variant is the one the fits perfectly the definition of the fuzzy set.

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